Quantum tic-tac-toe: A teaching metaphor for superposition in quantum mechanics

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Quantum tic-tac-toe was developed as a metaphor for the counterintuitive nature of superposition exhibited by quantum systems. It offers a way of introducing quantum physics without advanced mathematics, provides a conceptual foundation for understanding the meaning of quantum mechanics, and is fun to play. A single superposition rule is added to the child's game of classical tic-tac-toe. Each move consists of a pair of marks subscripted by the number of the move ("spooky" marks) that must be placed in different squares. When a measurement occurs, one spooky mark becomes real and the other disappears. Quantum tic-tac-toe illustrates a number of quantum principles including states, superposition, collapse, nonlocality, entanglement, the correspondence principle, interference, and decoherence. The game can be played on paper or on a white board. A Web-based version provides a refereed playing board to facilitate the mechanics of play, making it ideal for classrooms with a computer projector.
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I. INTRODUCTION

Quantum tic-tac-toe was developed in part to provide both a metaphor and an interactive activity for students to grapple with the weirdness of the quantum world. It requires neither mathematical training nor experimental apparatus of any kind. Although it can be played on paper or on a whiteboard, the most effective medium is a Web-based refereed board.[1]

Quantum tic-tac-toe is a variation on classical tic-tac-toe that formally adds only one rule, a rule of superposition. The paper consists of three main sections: rules, play, and metaphors. The rules section introduces superposition and the concepts that directly derive from it. Section III elaborates on common aspects and situations that result during the course of play. Section IV explores the features of the quantum world that quantum tic-tac-toe mimics and constitutes the heart of the paper.

The intent of this paper is to provide an introduction to quantum tic-tac-toe in sufficient depth for classroom use. The teacher can mix and match lectures, play, and challenges as appropriate for the students' level and coursework. Our expectation is that many students will advance quickly in their understanding of quantum tic-tac-toe and begin contributing observations to the class sometimes ahead of their teacher.

II. THE RULES

Quantum tic-tac-toe adds just one rule to the venerable child's game of classical tic-tac-toe, a rule of superposition. On every move, two marks must be placed in separate squares. These two marks are subscripted with the number of the move so X gets the odd number moves $\{(X_1, X_1), (X_3, X_3)\ldots\}$ and O gets the even number moves $\{(O_2, O_2), (O_4, O_4)\ldots\}$. These pairs of marks are called "spooky" marks in allusion to Einstein's comment about how the nonlocality of matter in quantum systems implies "spooky action at a distance." The quantum tic-tac-toe board is drawn with the dividing lines doubled (to reinforce the idea that play requires a pair of spooky marks) and with the squares numbered from 1 to 9. Quantum moves are indicated with hyphens, 1–3, 6–9,… Classical boards are smaller with single lines, unnumbered squares, and utilize only unsubscripted marks.

Superposition in quantum systems seems to imply that an object can be in two places at once, but only when we are not looking at it! Whenever we do look, that is, whenever a measurement is performed, a particle always ends up in only one place. The act of measurement yields classical values, not quantum ones. To predict the pattern of observations, the formalism of quantum mechanics implies the particle was in two places at once before we looked.

Quantum tic-tac-toe provides superposition with an immediate and obvious interpretation. Figure 1 shows the first move of a game where X places his spooky marks in squares 1 and 2. A superposition in quantum tic-tac-toe means that we are really playing two games of classical tic-tac-toe at once. In the first classical game X has moved to square 1, in the second classical game X has moved to square 2. The two classical games are in simultaneous play; they are not independent. Together they are called the classical ensemble and are isomorphic with the state of the game on the quantum board.

If superposition really means simultaneous classical games, what happens to the classical ensemble when O makes her move? Figure 2 shows the situation if she places her spooky marks in squares 4 and 5. The existing two classical games are duplicated so there are two identical sets, each consisting of two games. In the first set (row 1) O has moved to square 4, in the second set (row 2) O has moved to square 5, so there are now four games in the classical ensemble.

We might suppose that each successive quantum move would double the number of games in the classical ensemble. The actual situation is more subtle, as can be seen by considering a slightly different move for O in which she plays in squares 5 and 2 (see Fig. 3). X already has a spooky mark in square 2, so at first glance this move looks like it might be illegal. However, such moves must be permitted, else by move five X could not place both of his spooky marks because every square would already be occupied. As before,

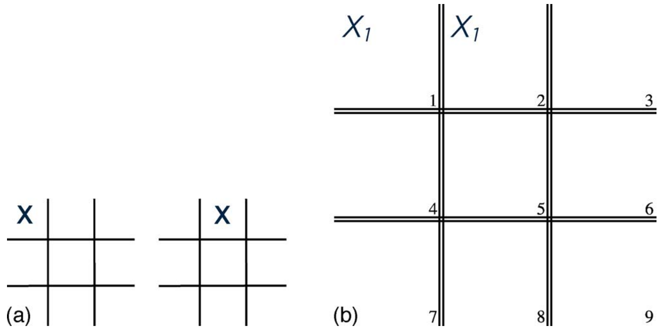

Fig. 1. Superposition implies a classical ensemble. (a) Classical ensemble. (b) The board in quantum tic-tac-toe is represented by doubled lines and numbered squares. Moves in quantum tic-tac-toe consist of a pair of spooky marks subscripted with the number of the move. Superposition implies a set of simultaneous classical tic-tac-toe games called the classical ensemble.

the classical games are duplicated into two sets with one of O's spooky marks as one move in the first set and her other spooky mark as the move in the second set.

This is a good time to point out that there is something peculiar about this classical ensemble and invite students to propose a resolution. The peculiarity is that in one of the classical games there is both an X and an O in square 2. These are real marks, not spooky marks, and such a position is illegal in classical tic-tac-toe. In quantum tic-tac-toe such games are eliminated from the classical ensemble (pruned) and they will be suppressed from now on.

With both X and O vying for square 2, the first two moves of the game are no longer independent. When a measurement finally occurs, if X ends up in square 2, O will have to end up in square 5. Similarly, if O ends up in square 2, X will have to be in square 1. These two moves have become entangled: what happens to one influences the other. Any pair of quantum moves that share a square are necessarily entangled.

With superposition and entanglement specified, we now consider the measurement problem, that is, how do spooky marks become real marks? Figure 4 shows a third move where X places his two spooky marks in squares 1 and 5 entangling with both moves one and two. All but two of the classical games in the ensemble are now contradictory, so pruning is particularly extensive. In addition, the resulting entanglement has a new property: it is possible to trace a

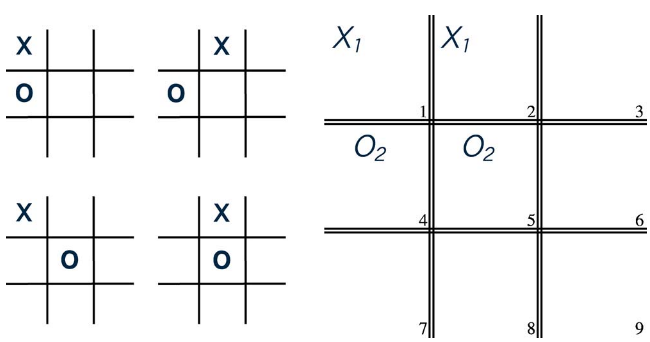

Fig. 2. Doubling the ensemble and separable states. Additional quantum moves replicate the exiting classical games in the ensemble into two sets. The first spooky mark represents the move in one set; the second spooky mark represents the move in the other set. O has doubled the size of the classical ensemble.

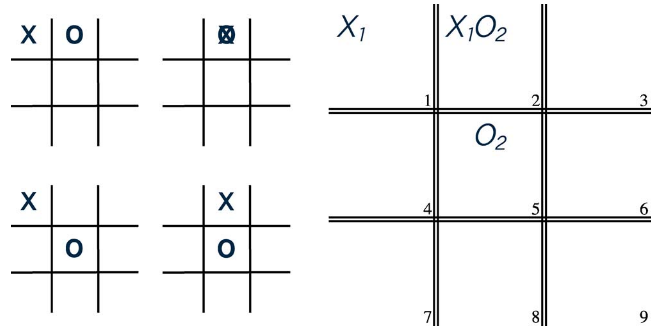

Fig. 3. Pruning the classical ensemble. Moves that place one spooky mark into a square already containing another spooky mark from a previous move lead to contradictory classical games. Contradictory games are pruned from the ensemble resulting in a nonseparable state and to entangling of the affected moves.

path around the entanglement back to the starting point. The entanglement is cyclic. Starting in square 2, X_1 is entangled with O_2 because they share the same square. However, O_2 is also entangled with X_3 in the center square, and similarly X_3 is entangled with X_1 in the corner square. If X_1 collapses to square 2, this collapse forces O_2 to collapse into square 5, forcing X_3 into square 1 which in turns forces X_1 into square 2, which is where this analysis started. Alternatively, if X_1 collapses to square 1, that forces X_3 into square 5, O_2 into square 2, and thus forces X_1 into square 1. If we assume a particular collapse for any of the moves, it creates a causal chain that forces the same collapse as was assumed. The entanglement is not only cyclic, it is self-referential. Cyclic entanglements will be indicated by underlining the spooky marks.

The instructor might wish at this point to introduce some basic concepts about formal systems and the difficulty they have with self-reference. Good examples are Russell's paradox from set theory, Cantor's diagonal argument about the transfinite nature of real numbers, circular arguments in logic, and the verbal liar's paradox, "This statement is false."[2]

None of the self-referential entanglements in quantum tic-tac-toe are paradoxical; they are all indeterminate—either collapse leads to a legal classical game. No matter how com-

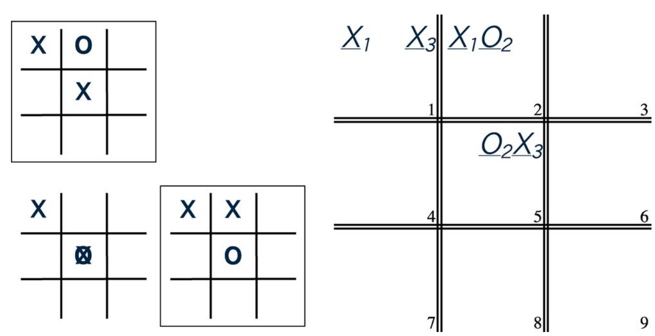

Fig. 4. Cyclic entanglement. Moves that have both spooky marks sharing squares with previous moves lead to cyclic entanglements and additional pruning of the ensemble. The resulting causal self-reference requires a collapse to classical values. The two valid classical games left in the ensemble are no longer in a superposition (indicated by their surrounding squares). Rather, one is to be chosen as the only one to go forward. The player who did not create the cyclic entanglement gets to make the choice, in this example; O gets to choose the collapse.

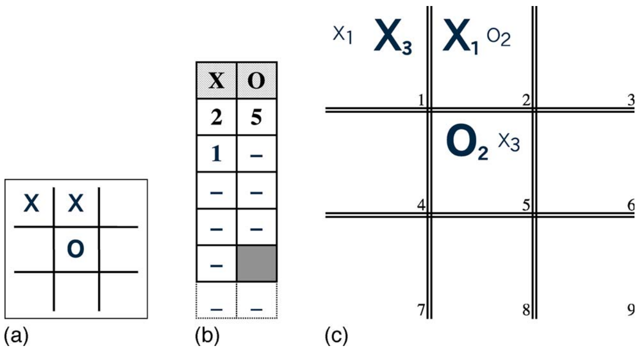

Fig. 5. Collapse to classical values. (a) Chosen classical game, (b) Listing of the classical moves, (c) Historical spooky marks. O chose the collapse that placed her move in the center square. It is now possible to record the moves of this sole remaining classical game. Move one was in square 2, move two in square 5, and move three in square 1. The spooky marks have been enlarged or diminished, respectively, to indicate the half kept and the half rejected. Self-reference has led to an objective measurement process.

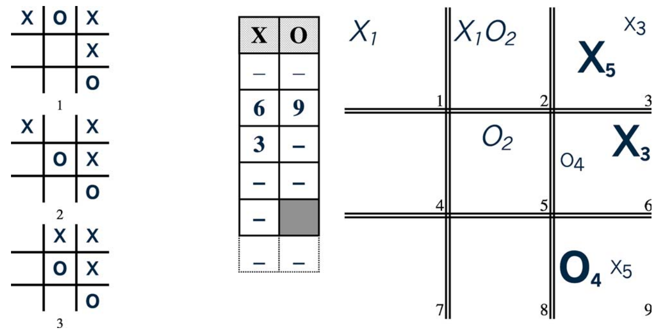

Fig. 6. Separate entanglements collapse separately. There are two entanglements on the quantum board. The one consisting of moves one and two is not cyclic (squares 1, 2, and 5). The entanglement of moves three, four, and five has already collapsed (squares 3, 6, and 9), leaving an incomplete listing and three classical games in the ensemble.

plicated the cyclic entanglement, there are always only two possible collapses. One of them must be chosen; the other will be eliminated from the classical ensemble, resulting in a single game of classical tic-tac-toe. To balance the game strategically, the other player gets to choose the collapse. Because X made the move that created a cyclic entanglement, O gets to choose the collapse. Afterward she will still get to make her regular quantum move of two spooky marks. Once a square has collapsed, it has a single real classical mark in it and further play in that square is prohibited.

As with any game, choosing a collapse represents a strategic choice. In this case, O can choose her move to end up either in the center square or the side square; the center square is tactically stronger so she chooses the collapse that puts her mark there. The other game in the classical ensemble is rejected as a matter of choice, not because of an entanglement-induced contradiction, and X and O now find themselves three moves into a classical game of tic-tac-toe (the only one left in the ensemble). Figure 5 shows the result after collapse along with a listing of the classical moves that correspond to the final reality.

In an experiment every measurement yields a single classical value; the quantum states behind it prior to the collapse (the spooky marks) are hidden from us. On the quantum board, the spooky mark of each pair that survives has been displayed in a larger font, and the spooky mark that has vanished has been displayed in a smaller font. This notation preserves the history of the quantum moves so students can confirm the cyclic entanglement and trace how the collapse occurred.

In quantum mechanics the measurement process is left unspecified. In contrast, quantum tic-tac-toe has an objective measurement process, but the particular measurement process used by quantum tic-tac-toe is less important than that there is one. One of the pedagogical strengths of quantum tic-tac-toe is that it highlights the issue of the measurement problem. This challenging topic will be revisited in Sec. IV on the physics metaphors provided by quantum tic-tac-toe.

III. PLAY

The addition of the rule of superposition to classical tic-tac-toe yields interesting behavior and suggests numerous metaphors with quantum physics. Before covering these metaphors in detail, we discuss some unexpected aspects of the game implied by this rule set. This discussion will reduce later confusion and strengthen the metaphors.

A. Separate entanglements

Figure 6 shows an incomplete quantum game that has suffered a cyclic entanglement on move five. There were two entanglements on this board. Moves one and two form a simple entanglement. The cyclic entanglement consisted of moves three, four, and five (rightmost column), which O has already collapsed. The principle here is that separate entanglements collapse separately. Although she had only two choices, O chose between two sets of realities in the classical ensemble with three classical games each. The one she chose gave the partial classical listing shown in Fig. 6 where moves one and two remain undetermined.

Because of the small size of the tic-tac-toe board, a maximum of four entanglements are possible. Every new quantum move (the pair of spooky marks) advances the game in one of four ways: it produces a new entanglement (all by itself), extends an existing entanglement, connects two existing entanglements making one larger one, or creates a cyclic entanglement requiring collapse.

Not all the moves involved in a cyclic entanglement are necessarily on the cycle itself. Those that are not (if any) are called *stems*. Regardless of which collapse is chosen for the cyclic part, the stems of an entanglement have only one possible collapse.

B. Winning

Figure 7 shows the successful conclusion of a game of quantum tic-tac-toe where O has achieved a winning 3-row down the middle column. A win requires three real classical marks in a row; spooky marks do not count. Any game that goes to nine moves without generating a classical 3-row is a cat's game (a tie). Note that if the board is fully collapsed on move eight without a win, then X is forced to place both his spooky marks in the sole remaining open square.

C. Simultaneous 3-rows

In a classical game, classical marks are added to the state of the board one at a time. In a quantum game classical

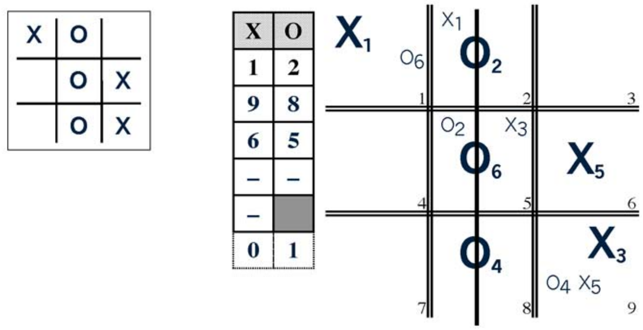

Fig. 7. A win requires a 3-row of classical marks. Here X chooses badly, and the collapse gives the game to O.

marks are added a group at a time. Each group was a single cyclic entanglement. Figure 8 shows a single entanglement that was just made cyclic by X. X has created a cycle between just two of his own moves (X_1 and X_7), so regardless of how O chooses the collapse, he will end up in both square 1 and square 5. Regardless of the collapse, X will have a 3-row in the first column, so his strategic position appears unassailable. In either case O will also get a 3-row in the third column. Should we regard these "simultaneous" 3-rows as a tie? Such ties are not possible in classical tic-tac-toe.

A look at the two classical games implied by each of the two possible collapses suggests a better alternative. Figure 8 shows both collapses and the listing of classical moves that corresponds to each. In the first game, X's first move collapsed into square 1 giving him a winning 3-row by move five of the game. O doesn't get her win until move six of the game (a move she would not even get to make in the classical game). Therefore, X is awarded a full point, but O is awarded a half point because she was able to delay X's win until she could also create a win for herself. In the second game, X's first move collapsed into the center square forcing his seventh move into square 1, so now he does not get his 3-row until move seven. O still gets her 3-row on move six so she is awarded the full point and X gets a half point for being just a little too late. In both cases, the moves that would be too late to happen in a classical game are shown in a shadow font in the listings, and as question marks on the classical boards.

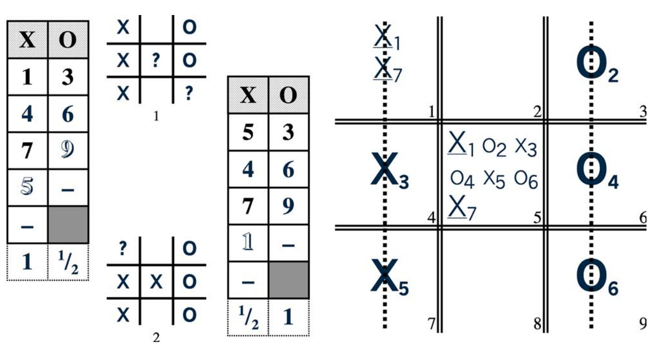

Fig. 8. In quantum tic-tac-toe, moves become classical in groups, instead of one-at-a-time as in classical tic-toe-toe. Therefore, it is possible for both players to get simultaneous 3-rows when a collapse occurs. However, the classical listing will reveal that one happened before the other. The earlier-in-time 3-row gets a full point, the later a half point. (Because X cyclically entangled with himself, both games in the ensemble will appear identical.)

That is not the end of the story. X made the move that created the cyclic entanglement, a move he thought was strategically sound because it gave him a classical 3-row regardless of the collapse. Nevertheless, O gets to choose the collapse. She should choose the second game, placing X's seventh move in the corner and winning by half a point.

There exists a set of games where all the permutations of these kinds of outcomes can be demonstrated. In some of them, X actually gets both 3-rows and can win with 1-1/2 points. It is left to the interested reader to determine if it is possible for X to get two classical 3-rows such that he wins with 2 points.

IV. PHYSICS METAPHORS

Although quantum tic-tac-toe is a fun game in its own right, the motivation for creating it was to make the nature of quantum mechanics more accessible. The transition from classical physics to quantum physics is difficult for many students and quantum physics appears to violate many preconceptions of common sense. Quantum tic-tac-toe offers one way to recondition those preconceptions.

The metaphors are organized into three topical areas: basic metaphors, advanced metaphors, and speculative metaphors.

A. Basic metaphors

Many of these metaphors have been alluded to in the presentation of the game. Here they are discussed in more detail and offered as a complete set.

1. States

In classical systems a property can take on a continuum of values. These values constitute the possible states of the system. In quantum systems the values of a property are in general quantized: they can have only discrete values. On the tic-tac-toe board, position is quantized: a mark must be in a square; its location in that square has no meaning and cannot span squares or lie between them.

In quantum mechanics a state is represented as a vector. In classical tic-tac-toe, there are nine possible states for a classical mark, each of the nine squares. Let the vector that represents a mark in square one be represented as $|1\rangle$; $|n\rangle$ represents a mark in the nth square. The state for a game is a concatenation of such vectors. For instance, if X plays first in the center, O plays in the diagonal on the lower right (square 9) and X responds in the first square, that game would be represented as

$$\Psi = |5\rangle_1 |9\rangle_2 |1\rangle_3. \tag{1}$$

The first move was in square 5, the second in square 9, and the third in square 1.

2. Superposition

With a little license, a pair of spooky marks can also be represented in Dirac notation. The move shown in Fig. 1 would be represented as

$$\Psi = \frac{1}{\sqrt{2}}\{|1\rangle_1 + |2\rangle_1\}. \tag{2}$$

Because superpositions in quantum tic-tac-toe are unweighted, the plus sign is suggestive, not literal; there are no coefficients associated with the base vectors. However, this notation is how an equally weighted superposition between two states would be represented mathematically in a quantum system (up to a phase angle).

The square root in Eq. (2) deserves special mention. In quantum mechanics it is called a normalization constant.[3] The square root indicates that there is a superposition of the amplitude, not the probabilities. To get probabilities we square the amplitudes, so an equally weighted superposition of two states has a 50/50 chance of turning out either way. Normalization constants are often suppressed in quantum calculations as a matter of convenience. In quantum tic-tac-toe, the number under the square root sign indicates how many classical games are in the classical ensemble.

3. Entanglement

There are two broad categories of states in quantum mechanics: separable states and nonseparable states. Figure 2 shows a separable state, which is the product of the states of the two moves on the board

$$\begin{aligned}
\Psi_X &= \frac{1}{\sqrt{2}}(|1\rangle_1 + |2\rangle_1) \\
\Psi_O &= \frac{1}{\sqrt{2}}(|4\rangle_2 + |5\rangle_2) \\
\Psi &= \Psi_X \Psi_O \\
\Psi &= \frac{1}{\sqrt{4}}(|1\rangle_1|4\rangle_2 + |1\rangle_1|5\rangle_2 + |2\rangle_1|4\rangle_2 + |2\rangle_1|5\rangle_2).
\end{aligned} \tag{3}$$

A nonseparable state is one that cannot be decomposed into product states. The situation shown in Fig. 3, where O's move is slightly different, is an example of a nonseparable state

$$\begin{aligned}
\Psi_X &= \frac{1}{\sqrt{2}}(|1\rangle_1 + |2\rangle_1) \\
\Psi_O &= \frac{1}{\sqrt{2}}(|2\rangle_2 + |5\rangle_2) \\
\Psi &\neq \Psi_X \Psi_O \\
\Psi &= \frac{1}{\sqrt{3}}(|1\rangle_1|2\rangle_2 + |1\rangle_1|5\rangle_2 + |2\rangle_1|5\rangle_2).
\end{aligned} \tag{4}$$

Both moves are in a two-way superposition, but the superpositions are not independent. Nonseparable states always imply an entanglement.

4. Transition to classical states

Like any quantum system, quantum states in quantum tic-tac-toe are converted to classical states when a measurement occurs.

5. Evolution/collapse duality

In quantum mechanics two processes affect the evolution of the system. One of these is specified by the Schrödinger equation. It specifies how a quantum system evolves in time when no measurements occur. The other is the measurement process itself, often called the reduction (or collapse) of the wave function. Penrose[4] has labeled these two processes U and R. U stands for unitary operator; a linear operator that modifies the direction of a vector in a Hilbert space but does not change its length. R stands for reduction; it is what happens to a vector when a measurement occurs.

These two processes could not be more different. U is deterministic; given the state U_0 at time zero, the state of the system can be predicted indefinitely into the future. R is nondeterministic. When a measurement occurs, the outcome is random with a probability that depends on the amplitude of the wave function associated with each possible result.

The metaphor with quantum tic-tac-toe is straightforward. There are two different kinds of moves in quantum tic-tac-toe: the placing of spooky marks mimicking U and the choice of a collapse when a cyclic entanglement occurs mimicking R. In quantum tic-tac-toe both of these moves are governed by the choice of the players, but it is not difficult to envision a variation where moves of the first type are generated automatically and deterministically by a computer, and the players only get to select collapses when a cyclic entanglement occurs. Even this last process could be given to a computer, utilizing a random number generator to make a nondeterministic choice.

For other quantum games, say, quantum checkers or quantum chess, the resulting games are too difficult for humans to play, so letting the computer evolve the superpositions until it is time for measurement might make such games viable. The players would alternate selecting collapses as a new measurement opportunity arises.

6. Correspondence principle

Because our everyday experience is of the classical world, the quantum nature of large systems must begin to emulate the classical behavior. This asymptotic behavior is called the correspondence principle. Imagine a game of quantum tic-tac-toe where collapse happened early and often. The board is filling up with classical marks. Although each new move must consist of a pair of spooky marks, as previous moves are collapsed, the board begins to look like a game of classical tic-tac-toe. The tic-tac-toe board is small, so the effect is not dramatic, but we can imagine a much larger board (as in Pente, a variation of tic-tac-toe played on a Go board) where the quantum nature is almost entirely masked by the transition to classical values.

B. Advanced metaphors

The following metaphors are not immediately obvious from the specification of the rules and correspond to more advanced topics in quantum mechanics. In many instances, these metaphors were found only by explicitly seeking game situations that exemplified them. There may remain others to be discovered.

1. Ascertainity principle

To demonstrate the ascertainity principle, we analyze a reasonably good opening in quantum tic-tac-toe and then analyze the classical game to which it collapses. In the example quantum play extends through move six when a cyclic entanglement occurs and a collapse is chosen. Figure 9 shows the state of the quantum board prior to the choice of

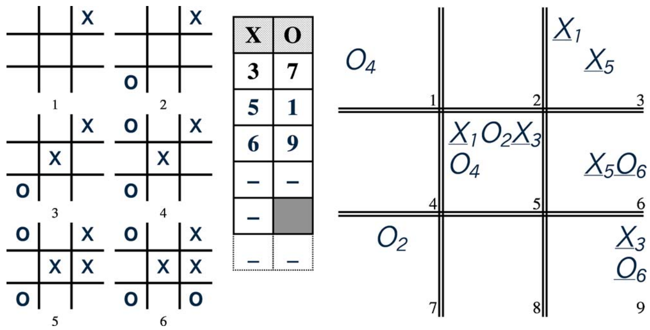

Fig. 9. Ascertainity principle. A good opening has just occurred on the quantum board resulting in a collapse to a single classical game. The listing of this game and its six-move history (not an ensemble) are shown to the left. Moves 2 and 3 are poor, moves 5 and 6 abysmal. The ascertainty principle asserts that given just the listing of classical moves, it is possible to ascertain whether a tic-tac-toe game was played under classical or quantum rules.

the collapse, the listing of classical moves of the collapsed game, and its classical history as a sequence of six classical boards.

First, consider the quantum game. X first moves in the second diagonal 3–5 which O defends 5–7 preventing him from ever getting a collapsed 3-row down this diagonal. X repeats this strategy in the first diagonal 5–9 and O responds similarly 1–5. Now X notices that he alone has spooky marks in the third column so he tries to capitalize on this advantage by playing both spooky marks in this column 3–6. O must respond, since X on his next move can force a collapse of these squares (by playing 3–5) winning with a 3-row in the third column. She chooses 6–9 creating a cyclic entanglement. X gets to select the collapse and he chooses the one that gives him an open 2-row in the second row. This series of moves is a reasonably strong opening for both players although alternatives are possible. Because this example is a bit complicated and somewhat difficult to follow from just a single static figure, a greater appreciation for it can be obtained by reviewing it on the game Web site,[1] where one can trace the moves forward and back with just a click of the mouse.

Now consider the resulting classical game. The listing of classical moves captures the data that was revealed when the quantum state of the board was measured. For clarity, the classical game is also laid out in Fig. 9 in the six classical boards to the left of the listing (not an ensemble). X's first move is fine, but O's response is poor; she needed to play in the center. Instead X plays there, a particularly poor move given that it results in a blocked 2-row for him; worse is that he had a winning sequence starting with either square 1 or 9. O's next move in the upper left corner is sound.

Four moves are now on the classical board and so it is X's move. Where should he play? Clearly in square 4 to block O's open 2-row in the first column, but instead he inexplicably plays in square 6. Where should O play? Also in square 4 to give her a 3-row and the game, but instead she even more inexplicably plays in square 9. What is going on? Four of the six moves in this classical game were poor, two of the four were catastrophic; that is, they failed either to generate or to prevent an immediate win.

The explanation is that the players were not playing classical tic-tac-toe but quantum tic-tac-toe. And the point is that given a listing of classical moves it is possible in general to ascertain whether the game was played under classical rules or under quantum rules. The ability to determine the "laws of the game" from only the classical outcomes is called the *ascertainity principle*. In the laboratory all we see are classical values; the quantum values are hidden from us—they must be inferred. Without the quantum values we cannot correctly predict the outcomes of experiments on quantum systems so we know they are there even though we cannot see them. The inability of classical physics to explain such experiments is why quantum theory was developed.

2. Quantum computing

The theoretical number of possible classical tic-tac-toe games can be found by counting all the permutations and is $9! = 362{,}880$. Humans are good at finding patterns and because the board is very symmetric and it takes several moves to break this symmetry, the actual number of possible unique games is more like 20,000.

The possibilities for quantum tic-tac-toe are much larger, about $9!^2$ or approximately 131 billion. This number is comparable to checkers but much less than for chess, although the branching ratio for quantum tic-tac-toe is about the same as for chess. This exponential increase in the game space is representative of what is expected for quantum computers. In principle, they should be capable of massively parallel operation, essentially computing many classical operations simultaneously. In this regard the classical ensemble provides an additional metaphor. Quantum computing is an active research field.[5] Getting a computer to play quantum tic-tac-toe is exponentially more difficult than getting one to play classical tic-tac-toe. And a quantum computer should be able to play quantum tic-tac-toe with about the same resources as a conventional computer would require to play classical tic-tac-toe.

3. Interference

Feynman has said that interference contains the only mystery of quantum mechanics.[6] In support of this claim, he discussed in detail the double slit experiment. In this experiment, a single particle is sent toward a pair of narrow slits beyond which lies a photographic screen that can record the location of a particle that makes it through. In the course of the experiment, many particles are sent through the slits. If only one slit is open, a broad diffraction pattern centered on the open slit is observed. If both slits are open simultaneously, the naive expectation might be that the resulting pattern would be the sum of the two diffraction patterns, an equally broad and smooth pattern centered between the slits. Instead, an interference pattern is obtained which is centered between the slits. The total intensity is the sum of the intensities of the two diffraction patterns but consists of bright and dark bands. There are specific locations on the screen (the center of each dark band) where the probability of detecting a particle is zero. If either slit were open, the particle has a nonzero chance of being detected at the center of a dark band. The mystery referred to by Feynman is how can the particle no longer be able to get to a spot when it is simply given another way to get there?

The availability of multiple paths to reduce options is not the end of the mystery, for if a means is concocted of detecting which slit the particle goes through, the interference pattern disappears and is replaced with the expected two overlapping diffraction patterns. These aspects are preserved

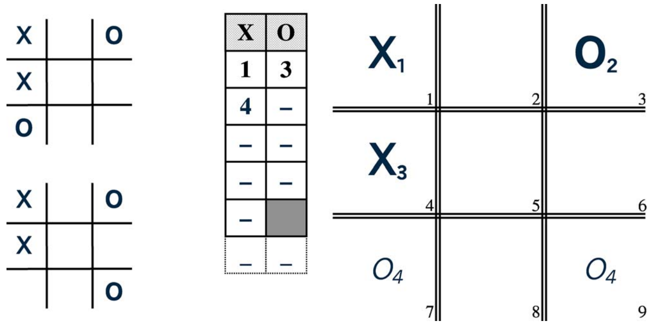

Fig. 10. Interference: classical game one. After the collapse of move three, O must respond to X's threat. Her best move gives her a 50/50 chance of ending up in either column 1 or column 3.

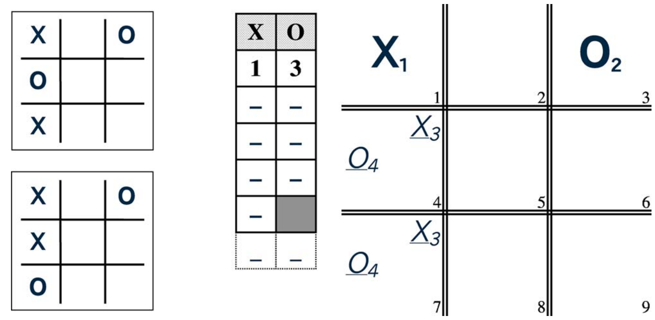

Fig. 12. Interference: superposition of classical games one and two. This game at move three is a superposition of the previous two games (Figs. 10 and 11). O's best move is now radically different, and she has no chance of her second move ending up in column three. The superposition of two classical realities have interfered with each other preventing an outcome otherwise permitted by either alone—just like the double slit experiment of physics.

even if the emission rate of particles is so low that only one particle at a time is between the source and the screen.

Amazingly, interference can be demonstrated in quantum tic-tac-toe. The idea is to consider two quantum games that have collapsed to similar classical games and then to consider the quantum game that is their superposition. Each classical game represents a case where one slit is open and the other closed; their superposition represents both slits open. These situations are shown in Figs. 10–12.

To minimize the number of figures, Figs. 10–12 each show the state of the game after O makes her best possible move (move four consisting of a pair of spooky marks). We will discuss the tactical situation on move three which leads to O's choice of move, so mentally subtract her quantum move while we analyze the board. Figure 10 shows the first of the three situations we will analyze. On move three of the game there were only three classical marks on the board (remember to ignore O's spooky marks) so a collapse must have happened on move three of the game. X had an open 2-row in the first column and was threatening to play one spooky mark in square 7 (the other one in any of the remaining six open squares). If O waits until move six to block this threat, say by making the same move as X did, the result would be a cyclic entanglement that X would get to collapse. X would choose the collapse that gave him the 3-row. If O does not try to block, X is free to make the same move again, only now regardless of how O collapses the cyclic entanglement, he gets his 3-row. Therefore, O must act now on move four.

She chooses squares 7 and 9; the reason for putting her second spooky mark in the third column is that if X proceeds with his threat, she makes nearly the same move, putting the second spooky mark in the other open square in the third column. Now if X self-collapses by repeating his previous move, even though he gets a 3-row in column one, he also gives O a 3-row in the third column, and because she can choose the collapse, she can insure that her 3-row is earlier in time, winning by a half point.

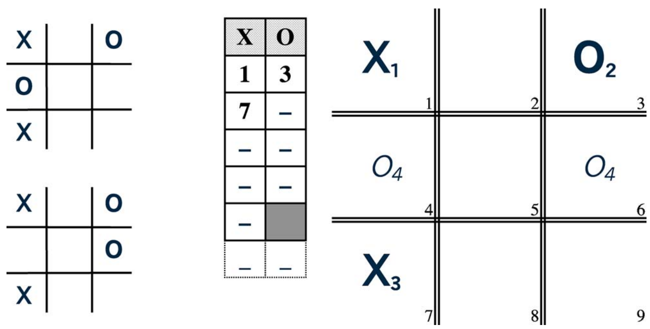

Fig. 11. Interference: classical game two. After the collapse of move three, O must respond to X's threat. Her best move gives her a 50/50 chance of ending up in either column 1 or column 3. This game is nearly the same as in Fig. 10, just shifted vertically.

Figure 11 shows the second classical game, which is almost the same as the first game; it is just that X's third classical mark has ended up in square 7 instead of square 4. X's threat is the same, just based off square 4 instead of square 7; O's best response is strategically the same, so she plays in squares 4 and 6. Her spooky marks have simply shifted position to align with the open square. In both of these classical games, O has a spooky mark in the first column and one in the third column. Without a strategic analysis, the most that can be said about these games is that she has an even chance of her move ending up in either column one or column three.

Now for the interference. Figure 12 shows a quantum game that is a superposition of the first two games. In this game the collapse occurred on move two, but the third move is still in a superposition and is arrived at by having each spooky mark placed in the two squares where the first two games differed. In this situation X's threat is much different; he is threatening to move into squares 4 and 7 on his next move, getting a 3-row by move five of the game. O's only possible response is to make this move first, placing both her spooky marks in the first column. She now has zero chance of her mark ending up in the third column. By having two possibilities, something that was possible with either alone has become impossible. The two classical realities have interfered with each other.

4. Nonlocality

In a famous series of debates with Niels Bohr about the foundations of quantum mechanics, Einstein devised the EPR thought experiment in 1935 with the help of Podolsky and Rosen.[7] In the EPR experiment a pair of quantum particles have become entangled so that measurements on them show such a high degree of correlation that measuring one would reveal exact information about the other. This inherent nonlocality appeared to offer a means to get around the

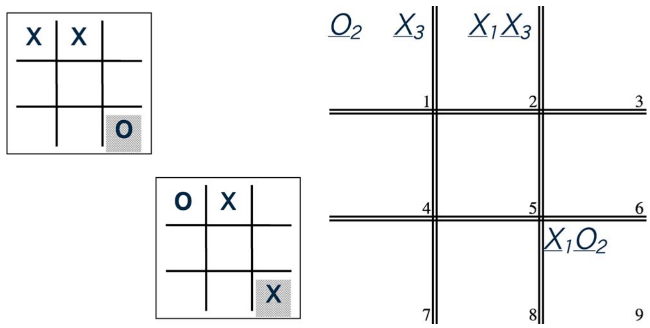

Fig. 13. Spooky action at a distance. The cyclic entanglement of move 3 in the upper left corner of the board has an immediate affect on the move that ends up in square 9 on the far corner of the board.

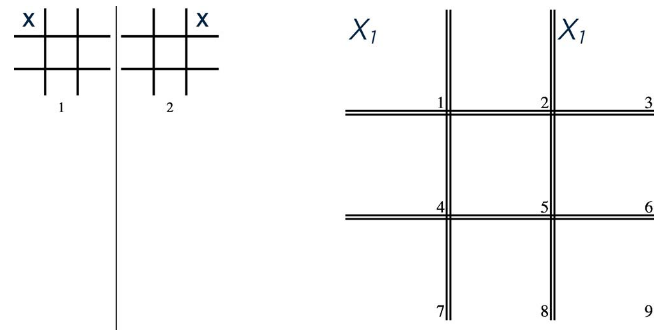

Fig. 14. Decoherence. An initial superposition has an equal chance of collapsing to either of two classical states (50%/50%).

Heisenberg uncertainty principle and was put forward by Einstein as an argument to show that quantum mechanics was incomplete.

Figure 13 shows an entanglement that spans opposite corners of the board. Move three (upper left hand corner) formed a cyclic entanglement. A portion of this entanglement is on the far side of the board in the lower right hand corner. The measurement triggered by the cyclic entanglement whose causative move was confined to one region of the board will affect whether an X or an O ends up in square 9, on the far side of the board. In particular, if there is an O in square 1, there will be an X in square 9, and if there is an X in square 1, there will be an O in square 9. The ability to influence regions of the board far from the actual move is an example of nonlocality, where a measurement on a portion of an entangled system has an instantaneous effect on another part of the system. Einstein called this "spooky action at a distance" because it seemed to violate the speed of light.[8] Because of the random nature of measurement, simple entanglements (such as an EPR pair) cannot be used to send superluminal information, but the correlations that exist between distant parts of an entangled system cannot be explained by either timelike causality nor a common cause in the past light cone.[9]

In 1969 John S. Bell published a proof that these correlations imply that quantum mechanics is nonlocal.[10] His theory was tested experimentally, first by Clauser,[11] and later by Aspect.[12] It has currently been demonstrated across a distance of 25 km by Gisin.[13]

5. Decoherence

Because the measurement mechanism is left unspecified in the current formulation of quantum mechanics, there are a number of ideas about the nature of the mechanism. One of these ideas is decoherence:[14] a quantum system becomes more and more classical as it becomes increasingly entangled with the environment. This concept can be demonstrated in quantum tic-tac-toe by considering the possibilities for the first move of the game as captured by the classical ensemble.

Figures 14–16 show what happens as additional random moves are played. The classical ensemble has been divided into two parts; the part on the left contains the games where the first move ends up in square 1, the part to the right contains the games where the first move ends up in square 3. As successive moves entangle with the first move, they form long entanglement chains off the spooky marks of move one. For random moves, any imbalance in the size of the two entanglement chains tends to grow, so the odds for the location of the first move very rapidly come to favor only one of its two possible locations.

In this example the first move of the game was in squares 1 and 3, which results in even odds for where move one will be. Move two entangled in square 1, so the odds are now 67% that move one will end up in square 3. Move three has one chance to entangle with the second spooky mark, but two chances to entangle with the first spooky mark, because it can either entangle directly with it or with move two. If it entangles with either, the odds now favor the first move ending up in square 3 by 75%. As the imbalance increases, the odds that it will increase further increase, so the situation diverges rapidly. Move four is now three times more likely to increase the imbalance even further, and if it does, move five will be four times as likely to increase it again. Figure 16 shows this situation, and the first move now has an 83% chance of ending up in square 3.

Decoherence remains a controversial topic, but the rapid descent to a classical value is in principle testable. Others, such as Penrose, believe that an objective measurement process is desired, required, and more likely.[15] The teacher might want to discuss several of the published proposals.[16–18] The measurement process in quantum tic-tac-toe is of the objective type and the example of decoherence in the game suggests that decoherence is an aspect of quantum systems as they approach the conditions necessary for collapse, but ontologically does not actually induce one.

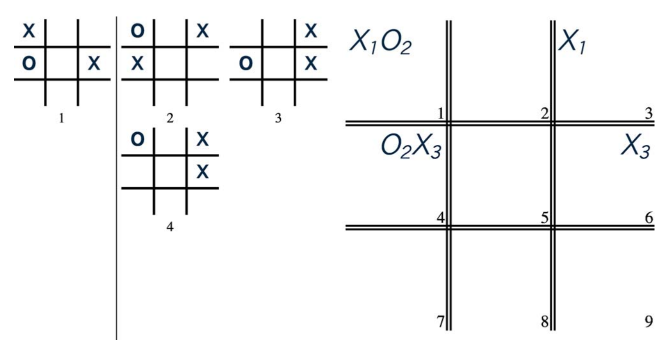

Fig. 15. Decoherence: growing entanglement. As it begins to entangle with its environment, the odds not only become unbalanced, but also favor becoming entangled with the environment in an unbalanced way (25%/75%).

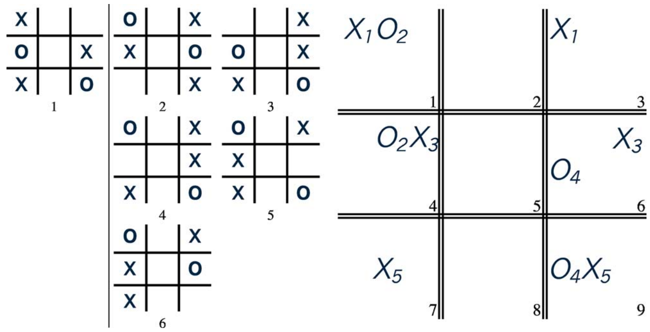

Fig. 16. Decoherence: entanglement unbalances rapidly. The unbalance in the environmental entangling diverges rapidly (17%/83%). The first move is far more likely to end up in square 3 than in square 1, and the odds are more likely to increase as further entanglements ensue.

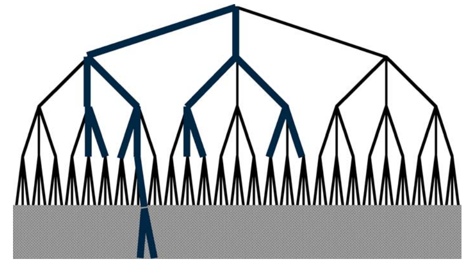

Fig. 17. Multipath vs single path through the classical game tree. Any one game in a classical game can be represented as a single path through the game tree. A quantum game has a game tree, a very much larger one in which it too is a single path, but a quantum game can also be mapped onto the game tree of its underlying classical game as a multibranched path with dead ends.

6. Many worlds

Because the measurement problem has gone unsolved for so long, some scientists are investigating ideas where collapse doesn't happen at all. One of these is the many worlds interpretation of quantum mechanics.[19] There appears to be at least two schools of thought about the meaning of the many worlds interpretation. One denies that collapse happens at all. Every time a measurement is made, the universe splits into as many copies as necessary, one for each possible outcome. Observers find themselves in a particular one of the copies, so it only looks like a collapse has happened. In this view the many worlds are all independent of each other. The major strength of this idea is that it makes collapse of the wave function a non-issue.

An alternative version of the many worlds interpretation has reality splitting upon superpositions. This version is closer to what happens in quantum tic-tac-toe with its classical ensemble. Its major strength is that it supplies a better epistemological explanation for interference. It is not clear that any of the many worlds interpretations are testable even in principle, which presents an opportunity to introduce students to the concept of metaphysics and the boundary between physics and philosophy.

One way that quantum tic-tac-toe can be a metaphor for the many worlds interpretation (at least the second version) is through the concept of the game tree. In a game tree, every node represents a possible state of the board, and all the lines emanating from that node represent legal moves. The first node is the initial state. In a classical game, any particular game is represented as a single path through the tree. A quantum game based on a classical game will have a vastly larger game tree. Recall from the discussion on quantum computing that a quantum game is exponentially larger than its classical counterpart. In the quantum game tree any particular quantum game is also represented by a single path. However, it is also possible to map a quantum game onto the much smaller classical game tree, but now any particular game is represented by a many-branched path as shown in Fig. 17. The multiple branches represent the many worlds, each branch representing one particular classical game. Note that pruning and collapse terminate some of these branches.

7. Nonlinearity of the measurement process

To understand the measurement problem, it is useful to ask what would be required to have an objective measurement process. In the current interpretation of quantum mechanics, we are in the uncomfortable position of having to admit that a measurement is one of those things where, "I know it when I see it." This ambiguity had the advantage of being practical and allowed the field to advance, but it is no longer very satisfying.

The primary requirement for an objective measurement process is that it be nonlinear. The Schrödinger equation, the projectors, and the unitary operators of Hilbert spaces are all linear. They predict how the wave function of an isolated system evolves and what the probabilities are when a measurement occurs, but they can't specify under what conditions a quantum system will collapse.

There are two ideas in this area worthy of note. One is the effort to add nonlinear terms to the Schrödinger equation;[20] the other is Penrose's suggestion that quantum gravity might supply the missing nonlinearity.[21] Both ideas have their difficulties. Adding nonlinear terms to the Schrödinger equation is an ad hoc approach and requires the existence of two new universal constants. Its primary benefit is that it is testable and perhaps in the near future can be tested. Because gravity is such a weak force, the possibility of testing gravity-induced collapse seems more remote. A more serious challenge is that knowing which test to try depends on the theory of quantum gravity of interest. Both theories are good examples of the role that imagination and ingenuity play in developing new theories.

Quantum tic-tac-toe supplies an objective measurement process that introduces nonlinearity through self-reference. This self-reference arises naturally from the nonlocal nature of superposition. Nonlocality is an uncomfortable concept because it implies spacelike causality and a particularly troubling type of self-reference, temporal paradox. The earliest effort to place self-reference and logical paradox on a firm theoretical foundation was developed by G. Spencer-Brown.[22] Self-reference is a many faceted concept and provides a good launching point for a discussion of linear versus nonlinear systems, mathematical tractability, and chaos theory, and reminds students that there are still areas of interest that are not well understood. For advanced students the teacher can introduce the topic of logical self-reference, including paradoxes and indeterminacies (circular arguments) and the role self-reference plays in major mathematical proofs such as Cantor's diagonal argument and Gödel's incompleteness theorem.[23]

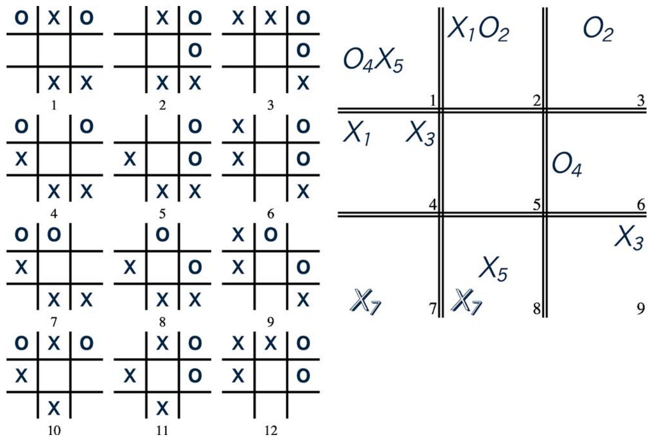

Fig. 18. Third type of causality. Through move five there are two entanglements leading to 12 realities in the classical ensemble. We are interested in the outcomes in squares 1 and 9 after O's third and fourth moves (moves 6 and 8), one of which will be either 9-6 or 9-8, the other either 1-2 or 1-4. In all cases, move 7 by X (shown in shadow font) will be the same.

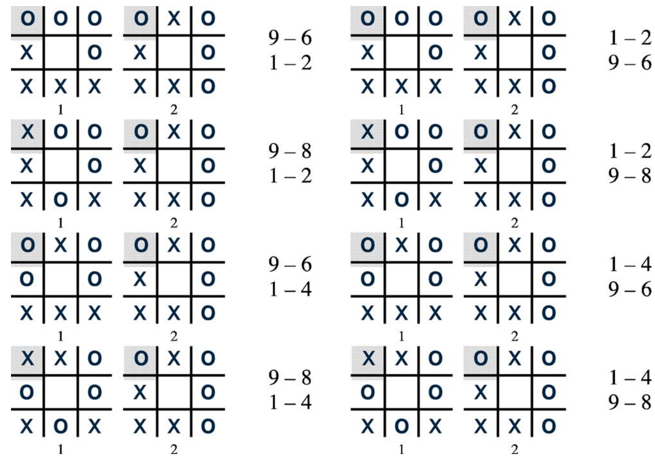

Fig. 19. Third type of causality: right causes left. There are 8 possible permutations for the two events; two choices each plus first and last. All produce two games in the classical ensemble. The outcome in square nine is always random, but square one depends on the right move, whether it is first (move 6) or last (move 8). Spacelike causality; right causes left, not earlier causes later.

C. Speculative metaphors

Although quantum tic-tac-toe offers a broad set of metaphors that are analogous to quantum physics, there are many aspects on which quantum tic-tac-toe is silent. This silence is to be expected because all that was added to the classical game was a superposition rule, and even it covers only unweighted superpositions. Complex weights are not applied. Also there is no concept of conjugate basis; the only basis set is the squares themselves, so there is no Hilbert space and no uncertainty principle. Quantum tic-tac-toe is played on a fixed space-time grid, so no relativity either.

However, quantum tic-tac-toe also suggests metaphors that go beyond our current understanding of quantum physics and should be regarded as speculative. Although not in the mainstream, these speculative metaphors are deserving of consideration because they highlight a part of the scientific enterprise that is typically given short shrift—imagination. Science is popularly characterized as rigid and subservient to the demands of experiment, which is the ultimate determiner of which theories are valid. However, the process of conceiving of theories is less structured and more an art in which imagination and speculation are essential skills.[24] So although these speculative metaphors must be taken with a grain of salt, they can be used to inspire, to stretch our imagination, and to remind us that science is still a recent human endeavor.

1. Third type of causality

Experiments have shown that timelike causality cannot explain the correlations seen in entangled quantum systems.[25] Bell's inequality shows that a common cause in the past light cone cannot explain them either. Nicolas Gisin has argued that there is apparently a new third type of causality that is required to explain the correlations seen in entangled quantum systems. Can we find such an example in quantum tic-tac-toe? Yes, but this case is more complicated than we have considered so far.

Figure 18 shows a game of quantum tic-tac-toe consisting of five uncollapsed quantum moves forming two separate entanglements. There are 12 realities in the classical ensemble. To cross entangle these two separate entanglements requires one quantum move, and to turn it into a cyclic entanglement (and thus lead to collapse) requires another. This example is clearer if both moves are by the same player (O in this case), so we will ignore the move X makes in between, always placing it in squares 7 and 8. To indicate this neglect, X's fourth move (move seven of the game) is shown in a shadow font, even though move six of the game has not yet been selected. The moves of interest are therefore moves six and eight, both to be made by O.

We are interested in treating the two moves by O as two events. One event will be on the left, centered on square 1, the other event will be on the right, centered on square 9. Each event (move) can be made in one of two ways. For the move on the left, it will be either 1-2, or 1-4, and for the move on the right, it will be either 9-6 or 9-8. The outcome of interest to this measurement is the classical results in square 1 and in square 9 and how they correlate with the chosen moves. Because there will be only two realities left in the classical ensemble, if the marks in squares 1 and 9 are the same between the two classical games, then the outcome is deterministic. If different, the outcome is random. Our goal is to demonstrate that a correlation exists between the outcomes and the choices and to see if the relation between them and the choices form a new type of causality, one that is neither timelike nor reducible to a common cause.

There are a total of eight permutations, a factor of 2 for each of the choices, and another factor of 2 to account for the order of the left and right moves. A summary of these permutations is shown in Fig. 19, where for each, the two realities in the classical ensemble are shown along with the moves of O that led to them. In all eight cases the outcome in square 9 is random, so neither choice nor order affects this outcome. However, the outcome of square 1 is more interesting and has been highlighted. Half the time it is always O, the other half it is random. Now for the correlation; it is random only when the move on the right was 9-8. There is no correlation with the move on the left; it is only the move on the far side of the board that determines the outcome in square 1. Counterintuitively, the move that square 1 is associated with, the move on the left, has no affect on the out-

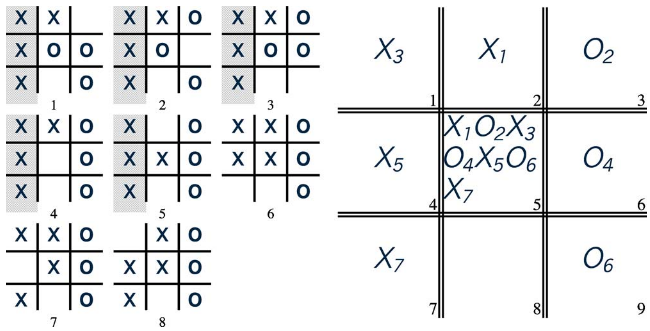

Fig. 20. Present influenced by futures that never happened. By move seven X has a win in five of the realities (1-5). O needs a move that will kill them all off.

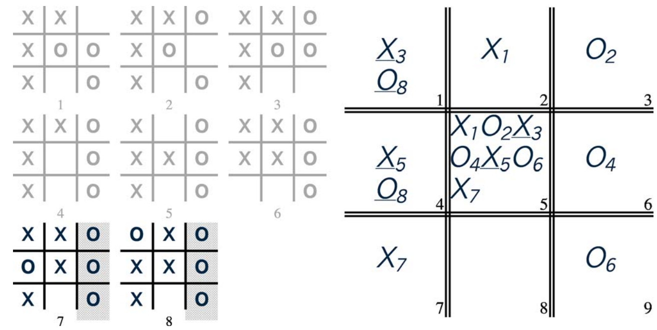

Fig. 21. Present influenced by futures that never happened: collapse. At time step seven, she detected an unfavorable event. She took action after this step, at time step eight, to make the undesirable event never happen, replacing it at time step six, before it never happened, with an outcome favorable to her; practical paradox-free time travel.

come of square 1. And it does not matter whether the move on the right occurs before the move on the left or after it! The "cause" of the outcome in square 1 is not the event that is earlier in time, but the event that is to the right. Here is a third type of causality, one that has some of the attributes of a spacelike cause.[26]

What accounts for this type of causality? At root it is just superposition. Superposition leads directly to the classical ensemble, and the classical games in the ensemble represent realities that interfere with each other. In addition, the realities pruned out of the ensemble when a collapse occurs represent information that gets lost. Because of pruning, realities that now no longer exist have still affected the present. When a game is over and fully collapsed to classical moves, the resulting history will provide no direct evidence of these pruned games, even though they had an effect on what transpired. From the point of view of the classical game to which the quantum game ultimately collapses, the present has been influenced by pasts that never happened.

Because collapse does not occur until an entanglement becomes cyclic, classical marks are added in groups-at-a-time instead of one-at-a-time as in a classical game. Because spooky marks become classical in groups, the perceived causality in the classical game arrived at by the collapse of a quantum state is spread out over multiple moves and is a general feature of quantum games. The cause that determined where a move finally ends, in which square it eventually takes on a classical value, is spread throughout a window that can be as wide as the entire game. The quantum move where the pair of spooky marks is placed begins this window; the collapse of the cyclic entanglement that includes it ends it. In quantum games, collapse is always backward in time. In a classical game, as in classical physics, the present is an infinitely thin barrier between the past and the future. In contrast, in a quantum game the present has a thickness; where the past ends and the future begins is not so well defined. As one wit put it, "In a classical game the future hasn't happened yet; in a quantum game neither has the present." Quantum tic-tac-toe entangles the near future with the recent past.[27]

Quantum mechanics is weird, but with the measurement problem still unresolved, perhaps it is not weird enough.[28] Quantum tic-tac-toe contains yet more surprises.

2. Influences by futures that never happened

We consider a situation where the near future and the recent past are inextricably entangled. Figure 20 shows a game of seven moves, where every move has placed one of its spooky marks in the center square. There are therefore eight games in the classical ensemble, and in five of them X has already won with a 3-row in the first column. O is faced with the need to eliminate realities where X has strong positions. Because he is strong in five of the eight realities, she has three possible moves.

Figure 21 shows her choice, and it eliminates all the realities where X had a win. But that is not the best part; in both the realities that remain, O has a 3-row in the third column, so no matter how X chooses the collapse, she will win the game. And even that is not the best part, because she wins in both of them by move six of the classical game to which this quantum game collapses. Her win, in classical time, is before X had any of his wins, none of which occurred until move seven.

Let us review. At time step 7 O detected an undesirable outcome. She took action after this step on time step 8 to make the undesirable outcomes go away, replacing them on time step 6 before they never happened with an outcome favorable to her. The present has been influenced by futures that once existed but now no longer do.[29]

The biggest surprise of quantum games is that to play them at the highest strategic level requires the players to consciously consider how the present move will change the past. All that was added to the classical game was a rule of superposition.

V. CONCLUSIONS

The metaphors provided by quantum tic-tac-toe form a foundation for introducing the counterintuitive nature of quantum physics. Quantum tic-tac-toe offers a means of breaking down many of the conceptual barriers that make quantum mechanics such a challenging subject, improving retention, and allowing these concepts to be presented to a wider audience. It has the potential to excite the imagination as well as to encourage students to consider how science advances and how science integrates the tension between speculation and rigor.

These pedagogical gains stem from adding just one rule to classical tic-tac-toe, a rule of superposition. One idea has generated a wealth of insight and understanding. In this way, quantum tic-tac-toe also shows what makes for a good physical theory. From the right key idea or unifying principle, a wealth of insight and understanding can emerge. This almost aesthetic sense is part of the expectation of what a good theory should look like. Have fun.

ACKNOWLEDGMENT

This work was supported by Novatia Labs.

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NOTHING COMES EASILY

Physicists spend a large part of their lives in a state of confusion. It's an occupational hazard. To excel in physics is to embrace doubt while walking the winding road to clarity. The tantalizing discomfort of perplexity is what inspires otherwise ordinary men and women to extraordinary feats of ingenuity and creativity; nothing quite focuses the mind like dissonant details awaiting harmonious resolution. But en route to explanation—during their search for new frameworks to address outstanding questions—theorists must tread with considered step through the jungle of bewilderment, guided mostly by hunches, inklings, clues, and calculations. And as the majority of researchers have a tendency to cover their tracks, discoveries often bear little evidence of the arduous terrain that's been covered. But don't lose sight of the fact that nothing comes easily. Nature does not give up her secrets lightly.

Brian Greene, *The Fabric of the Cosmos: Space, Time, and the Texture of Reality* (Knopf, 2004), p. 470.