GEOMETRY

Optativa 1r d'ESO

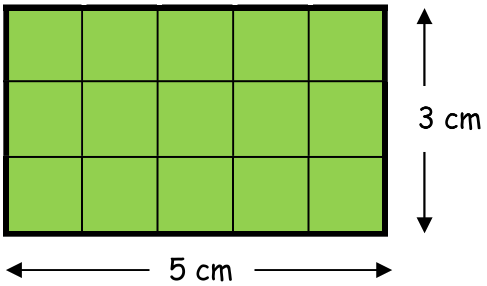
Dossier d'estiu per a recuperar la matèria al setembre. S'haurà d'entregar el dia de l'examen. L'examen valdrà un 50% i aquest dossier l'altre 50%.

S'han d'escriure tots els procediments.

AREAS AND PERIMETERS

1.- AREA AND PERIMETER

Look at the following rectangle. Its sides are 5 and 3 cm.



Remember that the **area** of a shape is the amount of space it covers.

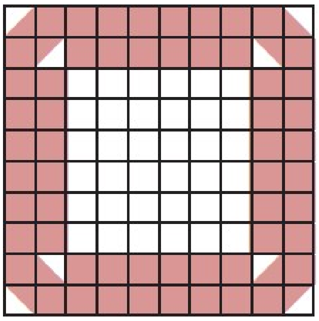
Since a **square centimetre** is the area of a square that is 1 cm on each side, the area of the rectangle above is $3 \cdot 5 = 15$ cm^2.

The **perimeter** of a plane shape is the total length around it.

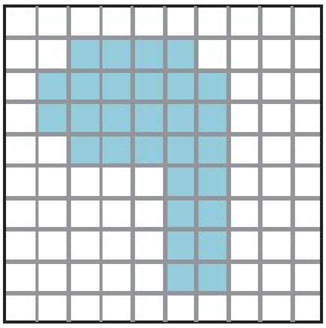
To get the perimeter of the rectangle above, add the lengths of all the sides, that is, is $3 + 3 + 5 + 5 = 16$ cm.

Exercise 1:

Determine the area of this picture.
Each shaded portion represents 1 square unit (1 sq unit).



Exercise 2:



Determine the area and the perimeter of this picture.
Each shaded portion represents 1 square unit (1 sq unit).

2.- AREA AND PERIMETER OF THE QUADRILATERALS

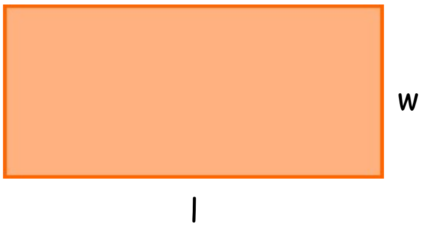
Square

s = length of side

$Area = s^2$

$Perimeter = 4s$

Rectangle

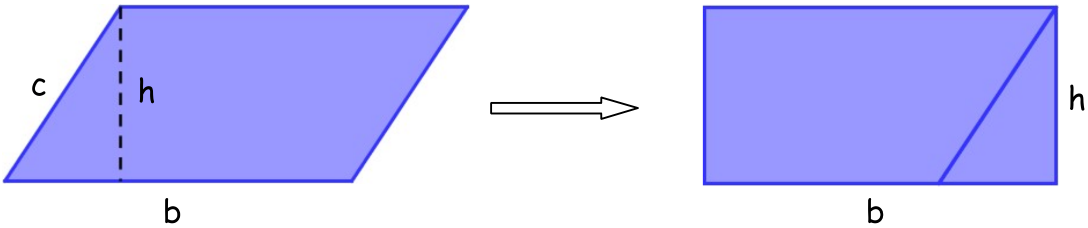


l = length   w = width

$Area = l \cdot w$

$Perimeter = 2l + 2w$

Parallelogram



If we move the triangle on the left to the right, the area of the rectangle that we get is exactly the same as the area of the parallelogram.

Therefore the area of the parallelogram will be $A = b \cdot h$.

b = base   h = height

$Area = b \cdot h$

$Perimeter = 2b + 2c$

Exercise 3:

A swimming pool was 10 m wide and 8 m long. What is the area of the pool?

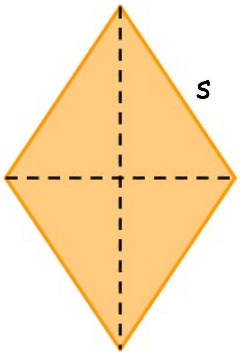
Exercise 4:

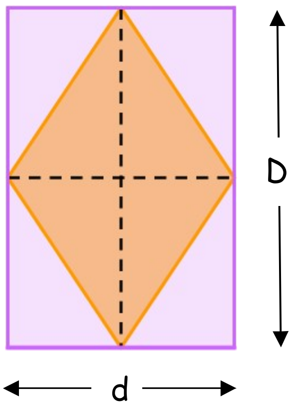
The area of a square field is 225 m^2. Find the perimeter of the field.

Exercise 5:

The perimeter of a rectangular carpet is 20 m. If the width of the carpet is 4 m, what is its length?

Rhombus



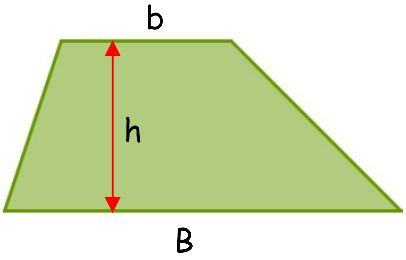


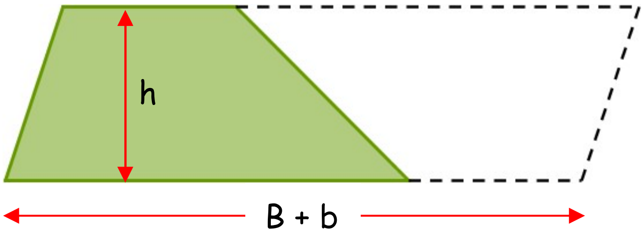
s = side of the rhombus
"D" and "d" are the diagonals of the rhombus

The area of the rhombus is one half of the area of the rectangle.

$$Area = \frac{D \cdot d}{2} \qquad Perimeter = 4s$$

Trapezium





h = height of the trapezium
"B" and "b" are the bases of the trapezium

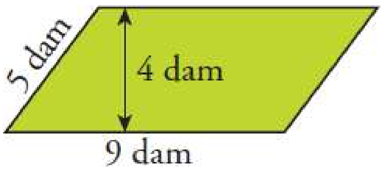
The area of the trapezium is one half of the area of the parallelogram whose base is "**B + b**" and whose height is "**h**".

$$Area = \frac{(B + b) \cdot h}{2}$$

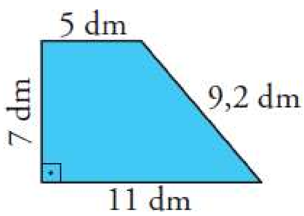
Exercise 6:

Find the area and the perimeter of the following quadrilaterals.

a)



b)



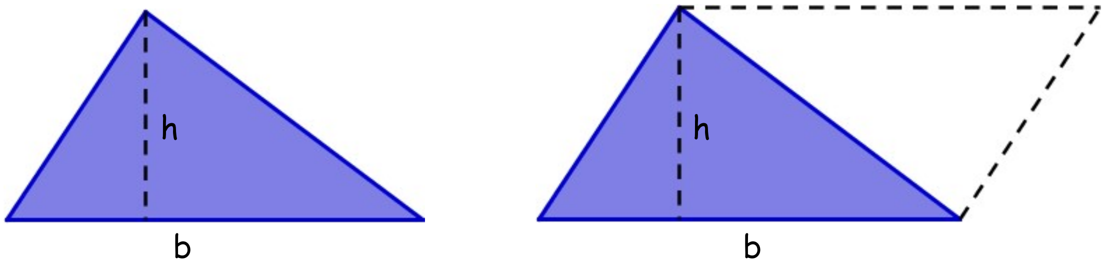
Exercise 7:

The diagonals of a rhombus are 37 and 52 cm. Find its area.

Exercise 8:

The diagonal of a square is 15 cm. Find its area.

3.- AREA OF A TRIANGLE

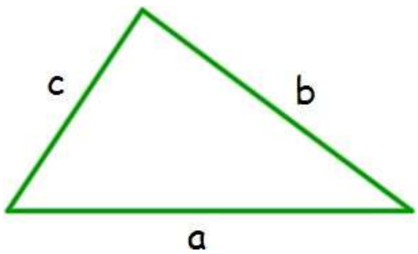


b = base of the triangle
h = height of the triangle

The area of the triangle is one half of the area of the parallelogram.

$$\text{Area} = \frac{b \cdot h}{2}$$

You can also calculate the area of a triangle if you know the lengths of all three sides, using a formula that is called **"Heron's Formula"** after Heron of Alexandria.



$$A = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}$$

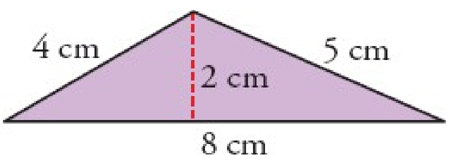
$$s = \frac{a+b+c}{2}$$

(s = semiperimeter of the triangle)

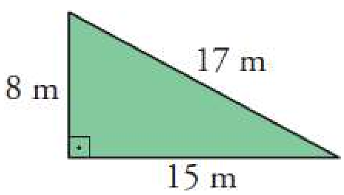
Exercise 9:

Find the area and the perimeter of the following triangles.

a)



b)



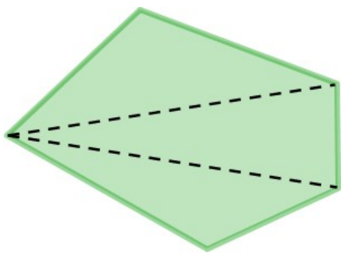
Exercise 10:

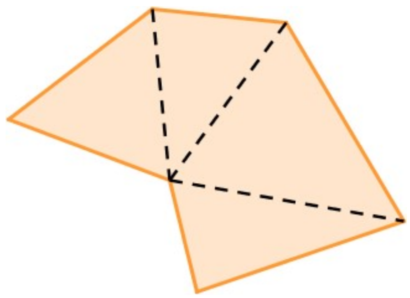
The three sides of a right triangle are 18, 24 and 30 cm.

a) Find the area of the triangle.
b) Find the length of the height to the hypotenuse.

4.- AREA OF A POLYGON

Look at these irregular polygons.



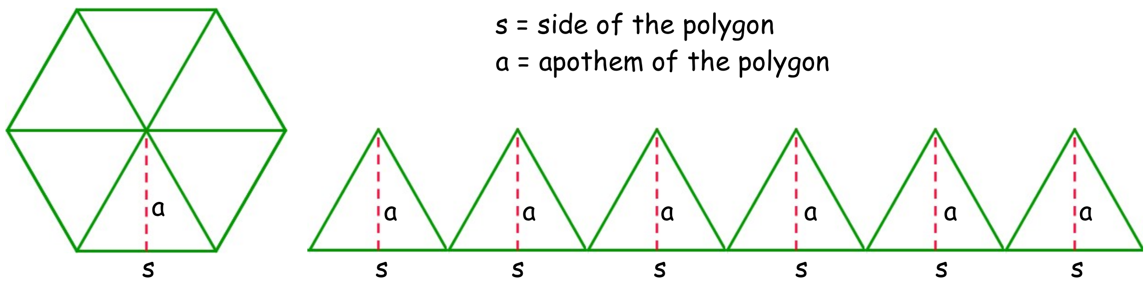


These polygons can be divided into triangles by drawing all the diagonals from one of the vertices. If you find the area of each triangle, then you can simply add them up to find the total area of the polygon.

Area of a regular polygon

A regular polygon of n sides can be divided into n equal triangles.

Look at these regular hexagon:



Since the 6 triangles are equal, the area $_s$of the polygon will be six times the area of the triangle, that is

$$Area = 6 \cdot \frac{s \cdot a}{2} = \frac{6s \cdot a}{2}$$

6s = perimeter of the polygon

In general, the area of a regular polygon is

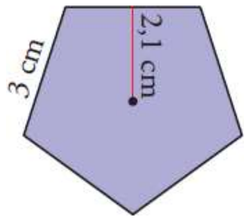
$$\text{Area} = \frac{\text{perimeter} \cdot \text{apothem}}{2}$$

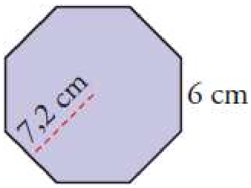
Exercise 11:

The side of a regular octagon is 15 cm, and its apothem is 18 cm. Find its area.

Exercise 12:

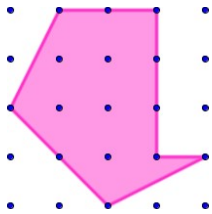
Find the area of the following regular polygons.

a) 

b) 

Exercise 14:

This shape is drawn on cm squared paper.
Calculate the area of the shaded shape



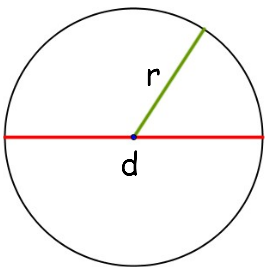
Exercise 13:

Use squared paper.

a) Draw three different triangles with an area of 8 sq units.
b) Draw three different parallelograms with an area of 12 sq units.
c) Draw three different trapezia with an area of 12 sq units.

5.- AREAS AND LENGTHS ON THE CIRCUMFERENCE

The set of points that are the same distance from a fixed point is a **circumference**.



The **diameter**, **d**, is the distance across the circle through the center.

The **radius**, **r**, is the distance from the center to the edge.

The length of the circumference is in direct **proportion** to its diameter: $C \approx 3d$

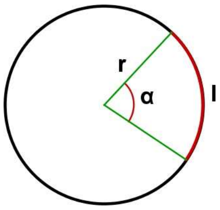
The actual proportion is a decimal. You use a symbol, π (pi).

$L = \pi \cdot d$ or $L = 2 \cdot \pi \cdot r$

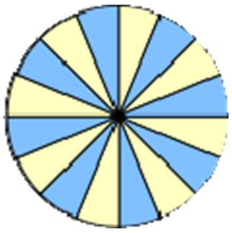
L: length of the circumference
d: diameter
r: radius
$\pi = 3.141592... \approx 3.14$

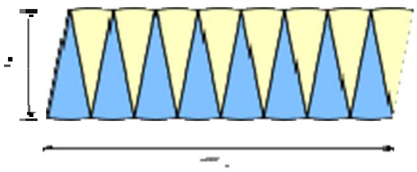
Exercise 15:

Prove that the **length of an arc** of a circumference that makes a central angle of α degrees is $l = 2 \cdot \pi \cdot r \cdot \frac{\alpha}{360}$.



You can cut a **circle** into lots of sectors and lay them out side by side, to make an approximate parallelogram.



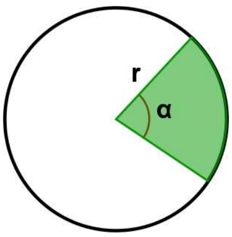


Area of the "parallelogram" $= \text{length} \cdot \text{width} = (\pi \cdot r) \cdot r = \pi \cdot r^2$

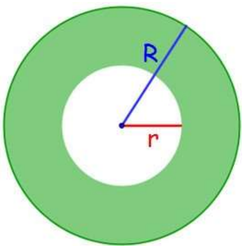
So the area of the circle is $A = \pi \cdot r^2$

Exercise 16:

Prove that the **area of a circle sector** that makes a central angle of α degrees is $A = \pi \cdot r^2 \cdot \frac{\alpha}{360}$.



Exercise 17:



Prove that the area of an **annulus** is $A = \pi \cdot (R^2 - r^2)$

Exercise 18:

A circular pond has radius 3 m. Work out the circumference of the pond.

Exercise 19:

A round hole has circumference of 44 cm. Work out the radius of the hole, to 1 decimal place.

Exercise 20:

A flowerbed in the park is semicircular. It has a radius of 2 m.

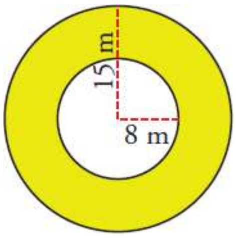
a) Work out the area of the flowerbed.

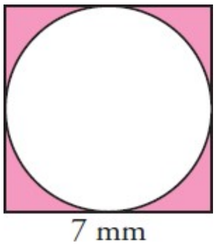
Percy the park keeper wants to plant flowers that each need an area of 0.3 m^2.

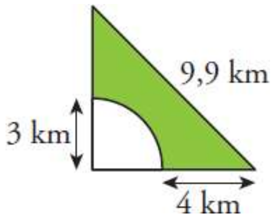
b) How many of these flowers can Percy plant in the flowerbed? What space does he have left?

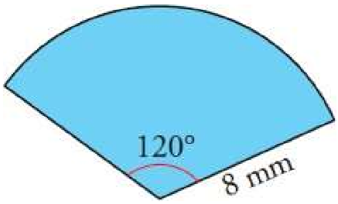
Exercise 21:

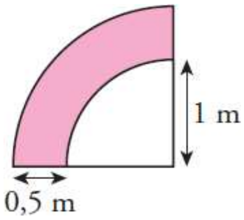
Find the area and the perimeter of the coloured shapes.

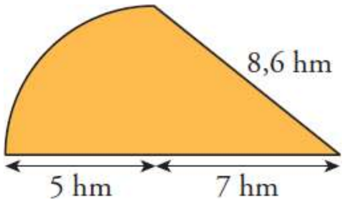
a) 

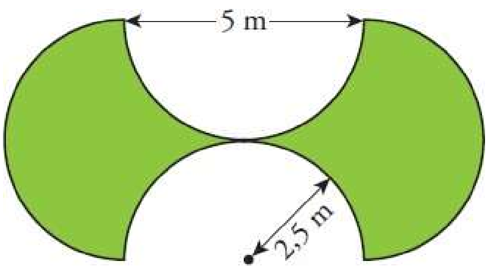
b) 

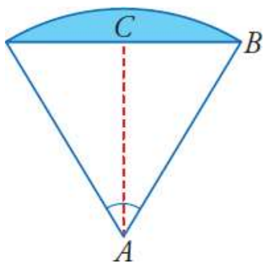
c) 

d) 

e) 

f) 

g) 

h) 

$\hat{A} = 60°$
$\overline{AB} = 10$ m
$\overline{AC} = 8.7$ m

MORE EXERCISES

1. Complete the following. Remember, you don't need to do any calculation. Just move the decimal point the appropriate number of places, and write the answer.

(a) 3 km ____________m **(b)** 4.5 m ____________cm

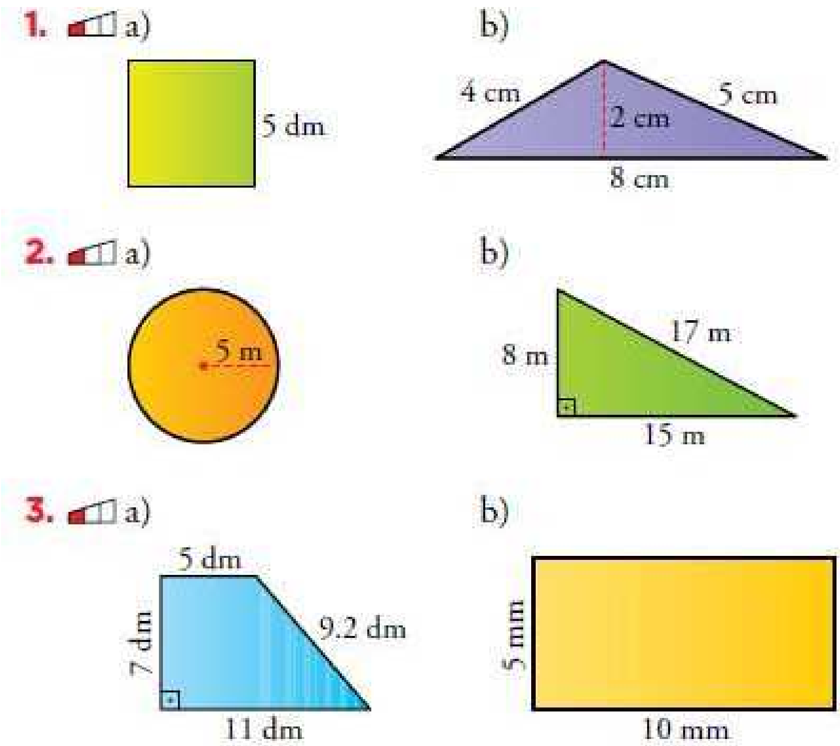
(c) 1.2 m ____________mm **(d)** 6.5 cm ____________mm

2. Complete:

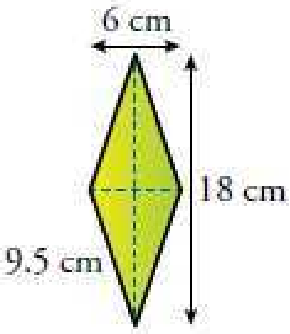
5 hm^2 = ... m^2 12 km^2 =.. dm^2

150 cm^2 = ...mm^2 61 dam^2 =.. km^2

3. Find the area and perimeter of each one of the coloured figures.

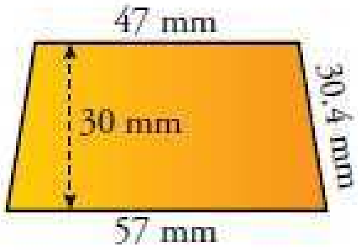


4. a)

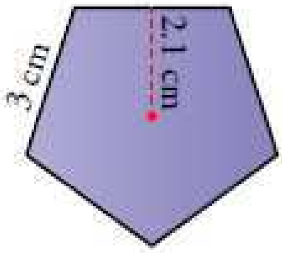


b)

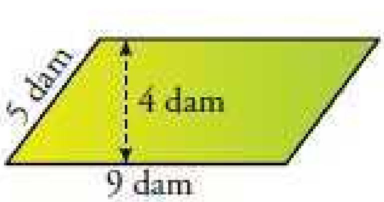
5. a)



b)



6. a)



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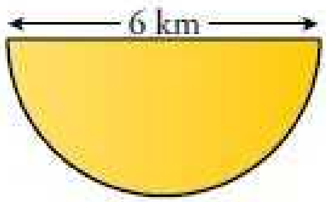

7. a)

b)